7-4 Remainder and Factor Theorems

Solve.
$x^3 + 4x^2 - 15x - 18 = 0$
If x - 3 is a factor.

3 | 1 4 -15 -18
3 21 18
1 7 6 | 0

$(x^2 + 7x + 6)(x - 3) = 0$
$(x + 6)(x + 1)$ $x = 3$
$x = -6$ $x = -1$
$\{-6, -1, 3\}$

Solve.
$x^3 + 7x^2 + 2x - 40 = 0$
If x - 2 is a factor.

x - c
2 | 1 7 2 -40
2 18 40
1 9 20 | 0
$(x^2 + 9x + 20)(x - 2) = 0$
$(x + 4)(x + 5)(x - 2) = 0$
$\{-4, -5, 2\}$

Solve.
$x^3 - 2x^2 + 9x - 18 = 0$
If x - 2 is a factor.

2 | 1 -2 9 -18
2 0 18
1 0 9 | 0
$(x^2 + 9)(x - 2) = 0$
$x^2 = -9$ $x = 2$
$x = \pm 3i$
$\{\pm 3i, 2\}$

When it is a factor, what can you say about the remainder?

Is it a factor?

$f(x) = x^3 + x^2 + 3x + 3$
Is $x + 2$ a factor?

$-2 \mid 1 \quad 1 \quad 3 \quad 3$
$\quad\quad -2 \quad 2 \quad -10$
$1 \quad -1 \quad 5 \quad -7$ No

$f(x) = x^3 + x^2 + 3x + 3$
Find f(-2).

$= (-2)^3 + (-2)^2 + 3(-2) + 3$
$-8 + 4 - 6 + 3$
-7

$f(x) = x^3 + x^2 + 3x + 3$
Is $x + 3$ a factor?

$-3 \mid 1 \quad 1 \quad 3 \quad 3$
$\quad\quad -3 \quad 6 \quad -27$
$1 \quad -2 \quad 9 \quad -24$
No

$f(x) = x^3 + x^2 + 3x + 3$
Is $x + 1$ a factor?

$-1 \mid 1 \quad 1 \quad 3 \quad 3$
$\quad\quad -1 \quad 0 \quad -3$
$\quad 1 \quad 0 \quad 3 \mid 0$

yes

Find k such that
$2x^4 + x^3 + 5x^2 - 6x + k \div x + 2$ has a remainder of 5.

$-2 \mid 2 \quad 1 \quad 5 \quad -6 \quad k$
$\quad\quad -4 \quad 6 \quad -22 \quad 56$
$\quad 2 \quad -3 \quad 11 \quad -28 \mid 56+k$

$56 + k = 5$
$k = -51$

$2x^4 + x^3 + 5x^2 - 6x + k$
Find k such that $x + 2$ is a factor.

$k + 56 = 0$
$k = -56$

Remainder Theorem (summary)
The remainder of $f(x) \div (x - a)$ is $f(a)$.

$f(x) \div (x + 2) = f(-2)$

Factor Theorem (summary)
The binomial $(x - a)$ is a factor of $f(x)$ iff $f(a) = 0$.

"if & only if"

$\therefore$ w/ & b/c

$f(x) = 3x^4 - 2x^3 + x^2 - 2$

f(4) =

f(2) =

$2\overline{)\,3 \quad -2 \quad 1 \quad 0 \quad -2}$

$\quad\quad\;\; 6 \quad 8 \quad 18 \quad 36$

$3 \quad 4 \quad 9 \quad 18 \quad (34)$

HW
p368-369
13-17, 21-27, 31-35 all odds